Surds, Indices & Proof

Q1.

Express as a single power of a

$$\frac{a^2}{\sqrt{a}}$$

where $a \neq 0$

Circle your answer.

a^1 $\quad$ $a^{\frac{3}{2}}$ $\quad$ $a^{\frac{5}{2}}$ $\quad$ a^4

(Total 1 mark)

Q2.

(a) Write $\sqrt[4]{x^3}$ in the form x^k.

(1)

(b) Write $\dfrac{1-x^2}{\sqrt[4]{x^3}}$ in the form $x^p - x^q$.

(2)

(Total 3 marks)

Q3.

The expression

$$\frac{3-\sqrt{n}}{2+\sqrt{n}}$$

can be written in the form $a + b\sqrt{n}$, where a and b and n are rational but $\sqrt{n}$ is irrational.

Find expressions for a and b in terms of n.

(Total 4 marks)

Q4.

Integers m and n are both odd.

Prove that $m^2 + n^2$ is a multiple of 2 but **not** a multiple of 4

(Total 5 marks)

Coordinate Geometry

Q5.

A circle with centre *C* has equation $x^2 + y^2 + 8x - 12y = 12$

(a) Find the coordinates of *C* and the radius of the circle.

(3)

(b) The points *P* and *Q* lie on the circle.
The origin is the midpoint of the chord *PQ*.
Show that *PQ* has length $n\sqrt{3}$, where n is an integer.

(5)

(Total 8 marks)

Q6.

A circle has equation

$$x^2 + y^2 - 6x - 8y = p$$

(a) (i) State the coordinates of the centre of the circle.

(1)

(ii) Find the radius of the circle in terms of p.

(3)

(b) The circle intersects the coordinate axes at exactly three points.

Find the **two** possible values of p.

(4)

(Total 8 marks)

Q7.

The line *L* has equation

$$3y - 4x = 21$$

The point *P* has coordinates (15, 2)

(a) Find the equation of the line perpendicular to *L* which passes through *P*.

(2)

(b) Hence, find the shortest distance from *P* to *L*.

(4)

(Total 6 marks)

Polynomials & Discriminant

Q8.

The equation $9x^2 + 4x + p^2 = 0$ has no real solutions for x.

Find the set of possible values of p.

Fully justify your answer.

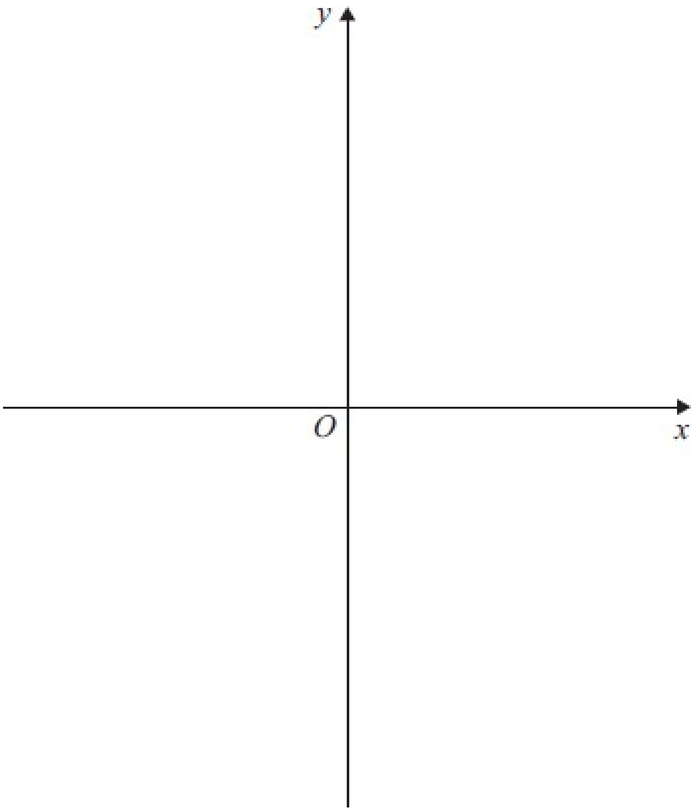
(Total 4 marks)

Q9.

(a) Sketch the curve with equation

$$y = x^2(2x + a)$$

where $a > 0$



(3)

(b) The polynomial $p(x)$ is given by

$$p(x) = x^2(2x + a) + 36$$

(i) It is given that $x + 3$ is a factor of $p(x)$

Use the factor theorem to show $a = 2$

(2)

(ii) State the transformation which maps the curve with equation

$$y = x^2(2x + 2)$$

onto the curve with equation

$$y = x^2(2x + 2) + 36$$

(2)

(iii) The polynomial $x^2(2x + 2) + 36$ can be written as $(x + 3)(2x^2 + bx + c)$

Without finding the values of b and c, use your answers to parts (a) and (b)(ii) to explain why

$$b^2 < 8c$$

(2)

(Total 9 marks)

Trigonometry

Q10.

(a) (i) Show that $\cos\theta = \frac{1}{2}$ is one solution of the equation

$$6 \sin^2 \theta + 5 \cos \theta = 7$$

(2)

(ii) Find all the values of θ that solve the equation

$$6 \sin^2 \theta + 5 \cos \theta = 7$$

for $0° \leq \theta \leq 360°$

Give your answers to the nearest degree.

(2)

(b) Hence, find all the solutions of the equation

$$6 \sin^2 2\theta + 5 \cos 2\theta = 7$$

for $0° \leq \theta \leq 360°$

Give your answers to the nearest degree.

(2)

(Total 6 marks)

Q11.

Find all the solutions of

$$9 \sin^2 x - 6 \sin x + \cos^2 x = 0$$

where $0° \leq x \leq 180°$

Give your solutions to the nearest degree.

Fully justify your answer.

(Total 4 marks)

Q12.

It is given that $(\tan \theta + 1)(\sin^2 \theta - 3 \cos^2 \theta) = 0$.

Find the possible values of $\tan \theta$.

(Total 4 marks)

Q13.

It is given that $5 \cos^2 \theta - 4 \sin^2 \theta = 0$

(a) Find the possible values of $\tan \theta$, giving your answers in exact form.

(3)

(b) Hence, or otherwise, solve the equation

$$5 \cos^2 \theta - 4 \sin^2 \theta = 0$$

giving all solutions of θ to the nearest 0.1° in the interval $0° \leq \theta \leq 360°$

(2)

(Total 5 marks)

Differentiation & Integration

Q14.

A curve has equation $y = \frac{2}{\sqrt{x}}$

Find $\frac{dy}{dx}$

Circle your answer.

$\frac{\sqrt{x}}{3}$ $\qquad$ $\frac{1}{x\sqrt{x}}$ $\qquad$ $-\frac{1}{x\sqrt{x}}$ $\qquad$ $-\frac{1}{2x\sqrt{x}}$

(Total 1 mark)

Q15.

It is given that $y = 3x - 5x^2$

Use differentiation from first principles to find an expression for $\frac{dy}{dx}$

(Total 4 marks)

Q16.

At the point (x, y) on a curve, where $x > 0$, the gradient is given by

$$\frac{dy}{dx} = 7\sqrt{x^5} - 4$$

(a) Write $\sqrt{x^5}$ in the form x^k, where k is a fraction.

(1)

(b) Find $\int \left(7\sqrt{x^5} - 4\right) dx.$

(3)

(c) Hence find the equation of the curve, given that the curve passes through the point (1, 3).

(3)

(Total 7 marks)

Q17.

A curve C has the equation

$$y = \frac{x^3 + \sqrt{x}}{x}, x > 0$$

(a) Express $\frac{x^3 + \sqrt{x}}{x}$ in the form $x^p + x^q$.

(3)

(b) (i) Hence find $\frac{dy}{dx}$.

(2)

(ii) Find an equation of the normal to the curve C at the point on the curve where $x = 1$.

(4)

(c) (i) Find $\frac{d^2y}{dx^2}$.

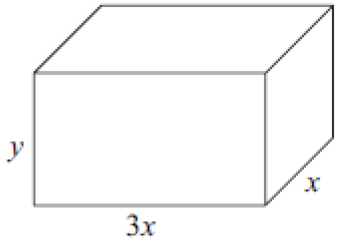
(2)

(ii) Hence deduce that the curve C has no maximum points.

(2)
(Total 13 marks)

Q18.

The diagram shows a solid cuboid with sides of lengths x cm, $3x$ cm and y cm.



The total surface area of the cuboid is 32 cm^2.

(a) (i) Show that $3x^2 + 4xy = 16$.

(2)

(ii) Hence show that the volume, V cm^3, of the cuboid is given by

$$V = 12x - \frac{9x^3}{4}$$

(2)

(b) Find $\frac{dV}{dx}$.

(2)

(c) (i) Verify that a stationary value of V occurs when $x = \frac{4}{3}$.

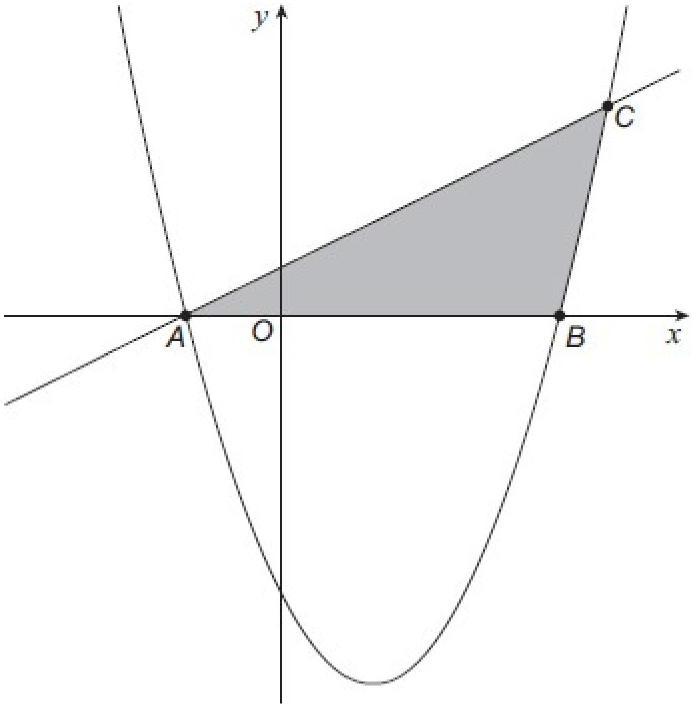
(2)

(ii) Find $\frac{d^2V}{dx^2}$ and hence determine whether V has a maximum value or a minimum value when $x = \frac{4}{3}$.

(2)
(Total 10 marks)

Q19.

The diagram below shows the graphs of $y = x^2 - 4x - 12$ and $y = x + 2$



(a) Write down three inequalities which together describe the shaded region.

(2)

(b) Find the coordinates of the points A, B and C.

(4)

(c) Find the exact area of the shaded region.

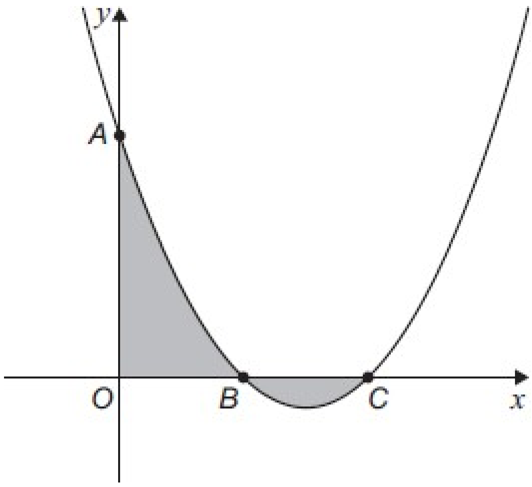
Fully justify your answer.

(6)

(Total 12 marks)

Q20.

The diagram below shows the graph of the curve that has equation $y = x^2 - 3x + 2$ along with two shaded regions.



(a) State the coordinates of the points *A*, *B* and *C*.

(2)

(b) Katy is asked by her teacher to find the total area of the two shaded regions.

Katy uses her calculator to find $\int_0^2 (x^2 - 3x + 2)\,dx$ and gets the answer $\frac{2}{3}$

Katy's teacher says that her answer is incorrect.

(i) Show that the total area of the two shaded regions is 1

Fully justify your answer.

(5)

(ii) Explain why Katy's method was not valid.

(1)

(Total 8 marks)

Exponentials & Logs

Q21.

(a) Given that $\log_a b = c$, express b in terms of a and c.

(1)

(b) By forming a quadratic equation, show that there is only one value of x which satisfies the equation $2 \log_2(x + 7) - \log_2(x + 5) = 3$.

(6)

(Total 7 marks)

Q22.

Show that the solution of the equation

$$5^x = 3^{x+4}$$

can be written as

$$x = \frac{\ln 81}{\ln 5 - \ln 3}$$

Fully justify your answer.

(Total 4 marks)

Q23.

Find the solution to

$$5^{(2x + 4)} = 9$$

giving your answer in the form $a + \log_5 b$, where a and b are integers.

(Total 3 marks)

Q24.

(a) Write each of the following in the form $\log_a k$, where k is an integer:

(i) $\log_a 4 + \log_a 10$;

(1)

(ii) $\log_a 16 - \log_a 2$;

(1)

(iii) $3 \log_a 5$.

(1)

(b) Use logarithms to solve the equation $(1.5)^{3x} = 7.5$, giving your value of x to three decimal places.

(3)

(c) Given that $\log_2 p = m$ and $\log_8 q = n$, express pq in the form 2^y, where y is an expression in m and n.

(3)
(Total 9 marks)